CAMBRIDGE ASSESSMENT

STEP Examiners' Report 2008

Mathematics

STEP 9465, 9470, 9475

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

General Remarks

There were around 850 candidates for this paper – a slight increase on the 800 of the past two years – and the scripts received covered the full range of marks (and beyond!). The questions on this paper in recent years have been designed to be a little more accessible to all top A-level students, and this has been reflected in the numbers of candidates making good attempts at more than just a couple of questions, in the numbers making decent stabs at the six questions required by the rubric, and in the total scores achieved by candidates. Most candidates made attempts at five or more questions, and most genuinely able mathematicians would have found the experience a positive one in some measure at least. With this greater emphasis on accessibility, it is more important than ever that candidates produce really strong, essentially-complete efforts to at least four questions. Around half marks are required in order to be competing for a grade 2, and around 70 for a grade 1.

The range of abilities on show was still quite wide. Just over 100 candidates failed to score a total mark of at least 30, with a further 100 failing to reach a total of 40. At the other end of the scale, more than 70 candidates scored a mark in excess of 100, and there were several who produced completely (or nearly so) successful attempts at more than six questions; if more than six questions had been permitted to contribute towards their paper totals, they would have comfortably exceeded the maximum mark of 120. While on the issue of the "best-six question-scores count" rubric, almost a third of candidates produced efforts at more than six questions, and this is generally a policy not to be encouraged. In most such cases, the seventh, eighth, or even ninth, question-efforts were very low scoring and little more than a waste of time for the candidates concerned. Having said that, it was clear that, in many of these cases, these partial attempts represented an abandonment of a question after a brief start, with the candidates presumably having decided that they were unlikely to make much successful further progress on it, and this is a much better employment of resources.

As in recent years, most candidates' contributing question-scores came exclusively from attempts at the pure maths questions in Section A. Attempts at the mechanics and statistics questions were very much more of a rarity, although more (and better) attempts were seen at these than in other recent papers.

Comments on individual questions

Q1 The first question is invariably intended to be a gentle introduction to the paper, and to allow all candidates to gain some marks without making great demands on either memory or technical skills. As such, most candidates traditionally tend to begin with question 1, and this proved to be the case here. Almost 700 candidates attempted this question, making it (marginally) the second most popular question on the paper; and it gained the highest mean score of about 14 marks.

There were still several places where marks were commonly lost. In (i), setting $(x_2, y_2) = (x_1, y_1)$ and eliminating y (for instance) leads to a quartic equation in x. There were two straightforward linear factors easily found to the quartic expression, leaving a quadratic factor which could yield no real roots. Many candidates failed to explain why, or show that, this was so. In (ii), the algebra again leads to two solutions, gained by setting $(x_3, y_3) = (x_1, y_1)$. However, one of them corresponds to one of the solutions already found in (i), where the sequence is constant, and most candidates omitted either to notice this or to discover it by checking. Another very common oversight – although far less important in the sense that candidates could still gain all the marks by going the long way round – was that the algebra in (ii) was *exactly the same* as that in (i), but with $a = -x$ and $b = -y$. For the very few who noticed this, the working for the second half of the question was remarkably swift.

Q2 Noticeably less popular than Q1 – with only around 500 "hits" – and with a very much poorer mean mark of about 8, it was rather obvious that many candidates were very unsure as to what constituted the *best* partial fraction form for the given algebraic fraction to begin with. Then, with very little direct guidance being given in the question, candidates' confidence seemed to ebb visibly as they proceeded, being required to turn the resulting collection of single algebraic fractions into series, using the *Binomial Theorem*, and then into a consideration of general terms. There was much fudging of these general terms in order to get the given answers of either $n + 1$ or $n + 2$ for the general term's coefficients; even amongst those who did spot which one occurred when, there was often little visible justification to support the conclusions. As a result of all the hurdles to be cleared, those who managed to get to the numerical ending successfully were very few in number.

Q3 This, the third most popular question on the paper, producing a mixed bag of responses. It strikes me that, although the A-level specifications require candidates to understand the process of *proof by contradiction*, this is never actually tested anywhere by any of the exam. boards. Nonetheless, it was very pleasing to see that so many candidates were able to grasp the basic idea of what to do, and many did so very successfully. The impartial observer might well note that the situation in (i) is very much tougher (in terms of degree) than that in (ii). However, candidates were very much more closely guided in (i) and then left to make their own way in (ii).

Apart from the standard, expected response to (i) – see the *SOLUTIONS* document for this – many other candidates produced a very pleasing alternative which they often dressed up as *proof by contradiction* but which was, in fact, a direct proof. It was, however, so mathematically sound and appealing an argument (and a legitimate imitation of a *p by c*) that we gave it all but one of the marks available in this part of the question. It ran like this:

Suppose w.l.o.g. that $0 < a \le b \le c < 1$.
Then $ab(1 - c) \le b^2(1 - b) \le \frac{4}{27}$ by the previous result
(namely $x^2(1 - x) \le \frac{4}{27}$ for all $x \ge 0$).
QED.

[Note that we could have used $ab(1 - c) \le c^2(1 - c) \le \frac{4}{27}$ also.]

It has to be said that most other inequality arguments were rather poorly constructed and unconvincing, leaving the markers with little option but to put a line through (often) several pages of circular arguments, faulty assumptions, dubious conclusions, and occasionally correct statements with either no supporting reasoning or going nowhere useful.

There was one remarkable alternative which was produced by just a couple of candidates (that I know of) and is not included in the *SOLUTIONS* because it is such a rarity. However, for those who know of the *AM – GM Inequality*, it is sufficiently appealing to include it here for novelty value. It ran like this:

Assume that $bc(1-a),\ ca(1-b),\ ab(1-c) > \frac{4}{27}$.
Using the previous result, we have $a^2(1-a),\ b^2(1-b),\ c^2(1-c) \le \frac{4}{27}$.
Then, since all terms are positive, it follows that $a^2 \le bc,\ b^2 \le ca,\ c^2 \le ab$ so that

$$a^2 + b^2 + c^2 \le bc + ca + ab. \quad \textbf{(*)}$$

However, by the *AM – GM Inequality* (or directly by the *Cauchy-Schwarz Inequality*),

$$a^2 + b^2 \ge 2ab\ ,\ \ b^2 + c^2 \ge 2bc \text{ and } c^2 + a^2 \ge 2ca\ .$$

Adding and dividing by two then gives $a^2 + b^2 + c^2 \ge ab + bc + ca$, which contradicts the conclusion **(*)**, etc., etc.

Q4 Another very popular question, poorly done (600 attempts, mean score below 7). Most efforts got little further than finding the gradient of the normal to the curve, and I strongly suspect that this question was frequently to be found amongst candidates' non-contributing scorers. Using the tan ($A-B$) formula is a sufficiently common occurrence on past papers that there is little excuse for well-prepared candidates not to recognise when and how to apply it. Once that has been done, the question's careful structuring guided able candidates over the hurdles one at a time, each result relying on the preceding result(s); yet most attempts had finished quite early on, and the majority of candidates failed to benefit from the setters' kindness.

Q5 This was the most popular question on the paper (by a small margin) and with the second highest mean mark (12) of all the pure questions. Those who were able to spot the two standard trig. substitutions $s = \sin x$ and $c = \cos x$ for the first two parts generally made excellent progress, although the log. and surd work required to tidy up the second integral's answer left many with a correct answer that wasn't easy to do anything much useful with at the very end, when deciding which was numerically the greater. The binomial expansion of $(a + b)^5$ was handled very comfortably, as was much of the following inequality work. However, the very final conclusion was very seldom successfully handled as any little mistakes, unhelpful forms of answers, etc., prevented candidates' final thoughts from being sufficiently relevant.

Q6 This was the least popular of the pure maths questions. Although there were 300 starts to the question, most of these barely got into the very opening part before the attempt was abandoned in favour of another question. Most attempts failed to show that f(x) has a period of 4π. As mentioned, few proceeded further. Of those who did, efforts were generally very poor indeed – as testified to by the very low mean mark of 4 – with the necessary comfort in handling even the most basic of trig. identities being very conspicuous by its absence. Part (iii) was my personal favourite amongst the pure questions, as it contained a very uncommon – yet remarkably simple – idea in order to get started on the road to a solution. The idea is simply this: f(x), being the sum of a cosine term and sine term, is equal to 2 *if and only if* each of these separate terms is simultaneously at its maximum of 1. That is, the question is actually two very easy trig. equations disguised as one very complicated-looking one. Once realised, the whole thing becomes very straightforward indeed, but only a few candidates had persevered this far.

Q7 In many ways, part (i) of the question was very routine, requiring little more than technical competence to see the differential equation, using the given substitution, through to a correct solution. Part (ii) then required candidates to spot a slightly different substitution on the basis of having gained a feel for what had gone on previously. I had thought that many more candidates would try something involving the square root of $1 + x^3$ or the cube root of $1 + x^2$, rather than cube root of $1 + x^3$, but many solutions that I saw went straight for the right thing. Once this had been successfully pushed through – with the working mimicking that of (i) very closely indeed – it was not difficult to spot the general answer required, unproven, in (iii). Overall, however, it seems that a lot of candidates failed to spot the right thing for part (ii) and their solutions stopped at this point. With almost 600 attempts, the mean score on this question was 10.

Q8 As with Q6, this was both an unpopular question and poorly done. Those candidates who did do well generally did so after spotting that they could use the *Angle Bisector Theorem* to polish off the first half of the question, expressing λ in terms of a and b almost immediately. Predominantly, the whole thing relied almost exclusively upon the use of the scalar product (or, alternatively, the *Cosine Rule*) and a bit of manipulation. The fact that the mean mark on this question was below 7 is simply indicative of the general lack of confidence amongst candidates where vectors are concerned.

Q9 Of the applied maths questions, this was by far the most popular, with over 400 attempts. However, most of these were only partial efforts, with few candidates even getting around to completing part (i) successfully, and the mean score ended up at about 8. Most candidates were comfortable with the routine stuff to start with, quoting and using the trajectory equation and using the identity $\sec^2\alpha = 1 + \tan^2\alpha$ to get a quadratic equation in $\tan\alpha$. For the remaining parts of the question, working was much less certain, even given the helpful information about small-angle approximations, and very few candidates were able to get a suitable approximation for $\tan\alpha$. Fewer still could turn an angle in radians into one in degrees.

Q10 Though much less popular than Q9, the attempts at this question followed a similar pattern, with most candidates coping pretty well with the routine opening demands – the use of the two main principles governing collisions questions: *Conservation of Linear Momentum* and *Newton's Experimental Law of Restitution* – but then falling down when a little more care and imagination were required in the parts that followed. With some careful application of ideas relating to similar triangles and a bit of inequalities work to follow, most candidates attempting these questions were just not up to the task. Few got as far as working on the initial and final kinetic energies; of these only a very small number noticed that there was a very quick way to go about it (see the *SOLUTIONS*). I don't recall seeing anyone successfully managing to get the right answer after having taken the longer route.

Q11 This question attracted under 100 attempts and a mean mark of under 3. The strong complaint I have made in the Report over recent years has consistently been that candidates' efforts on such questions have been seriously compromised by a disturbing inability to draw a decent diagram at the outset. I'm afraid that this was a major stumbling-block to successful progress with this question this year also. It was also a bit of a problem that candidates tended to confuse the acceleration of *P* relative to the wedge with its absolute acceleration relative to the stationary surface on which the wedge stood (say). As few decent attempts were made, it is difficult to be very specific about what went on otherwise.

Q12 There were almost 200 attempts to Q12, and the mean score was the highest – at 14 – of all the applied questions. This was partly due to the fact that the result of the first part could be largely circumvented by anyone who knew a little bit about expectation algebra, enabling them to write down $E(X)$ straightaway. The simple combinations of events, and their associated probabilities, in the final part of the question were very confidently and competently handled by most candidates and many polished the question off in its entirety relatively quickly.

Q13 Perhaps encouraged by the ease with which they had managed Q12, many of these candidates went on to attempt this question also. Although the listing of relevant cases was a fairly straightforward exercise, the handling of the binomial coefficients – which certainly *looked* clumsy and unappealing – was coped with much less well, and many mistakes were made in the ensuing algebra. In the very final part of the question, the idea that the calculus could lead to a nice, neat answer $\left(k = \sqrt{n(n-1)}\right)$ that then needed to be interpreted in terms of integer values, was just one step too far for most takers. The eventual mean score of 8 on this question testifies to the difficulties found in the algebra by most of the candidates who attempted it.